**IIT-JEE-Mathematics-Paper1-2007**

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**1.** Let α, β be the roots of the equation x2 - px + r = 0 and α/2, 2β be the roots of the equation x2 - qx + r = 0. Then the value of r is

(A) 2/9 (p - q)(2q - p)
(B) 2/9 (q – p)(2p – q)
(C) 2/9 (q – 2p)(2q – p)
(D) 2/9 (2p - q)(2q - p)

**2.** Let f(x) be differentiable on the interval (0, ∞) such that f(1) = 1, and

                       lim t->∞(t2 f(x)-x2 f(t))/(t-x) - 1

for each x > 0. Then f(x) is

(A) 1/3x + (2x2 )/3
(B) (-1)/3x + (4x2 )/3
(C) (-1)/x + 2/x2
(D) 1/x

**3.** One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to this wife is

(A) 1/2
(B) 1/3
(C) 2/5
(D) 1/5

**4.** The tangent to the curve y = ex drawn at the point (e, ee) intersects the line joining the points (e - 1, ee-1) and (e+1, ee+1)

(A) on the left of x = e
(B) on the right of x = e
(C) at no points
(D) at all points

**5.**      limx->π/4 ∫2sec2 x f(t) dt / ( x2 - π2/16 ) equals

(A) 8/π f(2)
(B) 2/π f(2)
(C) 2/π f(1/2)
(D) 4f(2)

**6.** A hyperbola, having the transverse axis of length 2 sin, is confocal with the ellipse 3x2 + 4y2 = 12. Then its equation is

(A) x2 cosex2 θ - y2 sec2 θ = 1
(B) x2 sec2 θ - y2 cosec2 θ = 1
(C) x2 sin2 θ - y2 cos2 θ = 1
(D) x2 cos2 θ - y2 sin2 θ = 1

**7.** The number of distinct real values of λ, for which the vectors -λ2î + ĵ + k̂, î - λ2ĵ + k̂ and î + ĵ - λ2k̂ are coplanar, is

(A) zero
(B) one
(C) two
(D) three

**8.** A man walks a distance of 3 units from the origin towards the north-east (N 45o E) direction. From there, he walks a distance of 4 units towards the north-west (N 45o W) direction to reach a point P. Then the position of P in the Argand plane is

(A) 3eiπ/4 + 4i
(B) (3 - 4i)eiπ/4
(C) (4 + 3i)eiπ/4
(D) (3 + 4i)eiπ/4

**9.**    The number of solutions of the pair of equations

                                2sin2 q - cos2q = 0

                                2cos2 q - 3 sin q - 0

        in the interval [0, 2p] is

        (A)    zero

        (B)    one

        (C)    two

        (D)    four

**10.**    Let H1, H2, ......, Hn be mutually exclusive and exhaustive events with P(Hi) > 0, i = 1, 2, ......, n. Let E be any other event with 0 < P (E) < 1.

        STATEMENT-1

        P(Hi|E) > P(E|Hi) • P(Hi) for i = 1, 2, ...... n.

        Because

        STATEMENT-2

        ∑ni=1 P(Hi) = 1.

(A)    Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B)    Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C)    Statement-1 is True, Statement-2 is False

(D)    Statement-1 is False, Statement-2 is True

**11.**   Tangents are drawn from the point (17, 7) to the circle x2 + y2 = 169.

        STATEMENT-1

        The tangents are mutually perpendicular

        Because

        STATEMENT-2

The locus of the points form which mutually perpendicular tangents can be drawn to the given circle is x2 + y2 = 338.

(A)    Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B)    Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C)    Statement-1 is True, Statement-2 is False

(D)    Statement-1 is False, Statement-2 is True

**12**.    Let the vectors PQ, QR, RS, ST, TU and UP represent the sides of a regular hexagon.

        STATEMENT-1

PQ-> × ( RS-> + ST-> ) ≠ 0->.

        Because

        STATEMENT-2

     PQ-> × RS-> = 0-> and PQ-> × ST-> ≠ 0->.

(A)    Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B)    Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C)    Statement-1 is True, Statement-2 is False

(D)    Statement-1 is False, Statement-2 is True

**13**.    Let F(x) be an indefinite integral of sin2x.

        STATEMENT-1

        The function F(x) satisfies F(x + p) - F(x) for all real x.

        STATEMTN-2

        sin2(x + p) = sin2x for all real x.

(A)    Statement-1 is True, Staement-2 is True, Statement-2 is a correct explanation for statement-1.

(B)    Statement-1 is True, Staement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C)    Statement-1 is True, Statement-2 is False

(D)    Statement-1 is False, Statement-2 is True

**14**.    The sum of V1 + V2 +......+ Vn is

        (A)    1/12 n(n + 1)(3n2 - n + 1)

        (B)    1/12 n(n + 1)(3n2 + n + 1)

        (C)    1/12 n(2n2 - n + 1)

        (D)    1/12 n(2n2 - 2n + 3)

**15**.    Tr is always

        (A)    an odd number

        (B)    an even number

        (C)    a prime number

        (D)    a composite number

**16**.    Which one of the following is a correct statement?

        (A)    Q1, Q2, Q3, ...... are in A.P. with common difference 5

        (B)    Q1, Q2, Q3, ...... are in A.P. with common difference 6

        (C)    Q1, Q2, Q3, ...... are in A.P. with common difference 11

        (D)    Q1 = Q2 = Q3 =......

**Paragraph**

        Consider the circle x2 + y2 = 9 and the parabola y2 = 8x. They intersect at P and Q in the first the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

**17**.    The ratio of the areas of the triangles PQS and PQR is

        (A)    1 : √2

        (B)    1 : 2

        (C)    1 : 4

        (D)    1 : 8

**18**.    The radius of the circumcircle of the triangle PRS is

        (A)    5

        (B)    3√3

        (C)    3√2

        (D)    2√3

**19**.    The radius of the incircle of the triangle PQR is

        (A)    4

        (B)    3

        (C)    8/3

        (D)    2

**20**.    Consider the following linear equations

                ax + by + cz = 0

                bx + cy + az = 0

                cx + ay + bz = 0

Match the conditions/expressions in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

|  |  |
| --- | --- |
| **Column I** | **Column II** |
| (A) |   a + b + c ¹ 0 and a2 + b2 + c2 = ab + bc + ca   | (p) | the equations represent planes meeting only at a single point. |
| (B) |   a + b + c = 0 and a2 + b2 + c2 = ab + bc + ca  | (q) | the equations represent the line x = y = z. |
| (C) |   a + b + c ¹ 0 and a2 + b2 + c2 ¹ ab + bc + ca  | (r) | the equations represent identical planes. |
| (D) |   a + b + c = 0 and a2 + b2 + c2 = ab + bc + ca  | (s) | the equations represent the whole of the three dimensional space. |

**21**.    In the following [x] denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties in Column II.

|  |  |
| --- | --- |
| **Column I** | **Column II** |
| (A) |   x|x| | (p) |   continuous in (-1,1) |
| (B) |   √|x| | (q) |    differentiable in (-1,1) |
| (C) |   x + |x| | (r) |   strictly increasing in (-1,1) |
| (D) |   |x - 1| + |x + 1| | (s) |   not differentiable at least at one point in (-1, 1) |

**22**.    Match the integrals in Column I with the values in Column II.

|  |  |
| --- | --- |
| **Column I** | **Column II** |
| (A) |  ∫-11 dx/(1+x2 ) | (p) |  1/2 log(2/3) |
| (B) |  ∫01 dx/√(1-x2 ) | (q) |  2log(2/3) |
| (C) |  ∫23 dx/(1-x2 ) | (r) |  π/3 |
| (D) |  ∫12 dx/(x√(x2-1)) | (s) |  -π/2 |